Homework – Incidence Geometry

Deadline: Mar. 12, 2013

1. Given a point $P = [x_0, y_0, z_0]$ and a line
$$\ell : ux + vy + wz = 0$$
in the real projective plane $\mathbb{P}$, where $(u, v, w) \neq (0, 0, 0)$ and P is not on ℓ. Find a homogeneous coordinate formula for the projection from P to ℓ. In particular, when P is the point at infinity on the x-axis and ℓ is the y-axis, explain your formula. (Show your steps)

2. Switch the names of points and lines in a projective plane $\boldsymbol{P}$ to have a new system $\boldsymbol{P}'$ of points and lines whose incidence are the same as in $\boldsymbol{P}$. Show that $\boldsymbol{P}'$ forms a projective plane.

3. Let $\boldsymbol{P}$ be a finite projective plane, i.e., there are finite number of points in $\boldsymbol{P}$. Show that every line has the same number of points. If each line has $n + 1$ points, show that
   (a) For each point in $\boldsymbol{P}$ there are exactly $n + 1$ lines through it.
   (b) The total number of points in $\boldsymbol{P}$ is $n^2 + n + 1$.
   (c) The total number of lines in $\boldsymbol{P}$ is $n^2 + n + 1$.

4. Let $\boldsymbol{P}$ be a projective plane and ℓ be a line in it. Delete all points on ℓ from $\boldsymbol{P}$ to have a subsystem $\boldsymbol{A}$ with points and lines, and keep the same incidence relation. Show that $\boldsymbol{A}$ forms an affine plane.